

# Towards the Construction of a Mathematical Model of an Induction Motor and Centrifugal Pump

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**ABSTRACT:** This paper presents a review of mathematical models of the most widely used electrical machines - squirrel cage induction motors. T-shaped and G-shaped substitution diagrams of induction motor are given. The equations of induction motor in rotating coordinate axes d, q are presented. Also, features of construction of mathematical model of centrifugal pump are given. When building the model, we used the centrifugal pump substitution diagram and the equation of the moment of resistance.

**KEY WORDS:** induction motor, replacement circuit, mathematical model, operating mode, centrifugal pump, mathematical model, substitution scheme.

## I. INTRODUCTION

It is well known that more than 60% of all electrical energy produced is generated by electric machines. Among electrical machines, asynchronous machines are the most widely used and are used in various applications. When considering methodological approaches to the construction of a mathematical model of induction machines, various approaches, methods of analysis, forms of mathematical description of static and dynamic modes of operation are used. The mathematical model should take into account all the main factors affecting the operating mode, including changes in inductive resistance due to saturation, active resistance due to the skin effect, resistance moment at steady state and transient operation of the electric drive.

Turbomachines are often used in mining, petrochemical, metallurgical, agricultural industries for transportation of various liquids and gaseous substances. Centrifugal pumps are one of the most common mechanisms with the highest electrical energy consumption. Consumption of electric energy for pumping installations in the Republic of Uzbekistan is more than 17% of all generated electric energy. This determines the need to save electric energy during transportation of various liquids.

## II. TO THE MATHEMATICAL MODEL OF AN INDUCTION MOTOR

To study the operating modes of induction motors, substitution diagrams are used to determine the basic relationships of individual quantities that characterise their operation under given conditions. Figure 1 shows substitution diagrams of an induction motor [1].

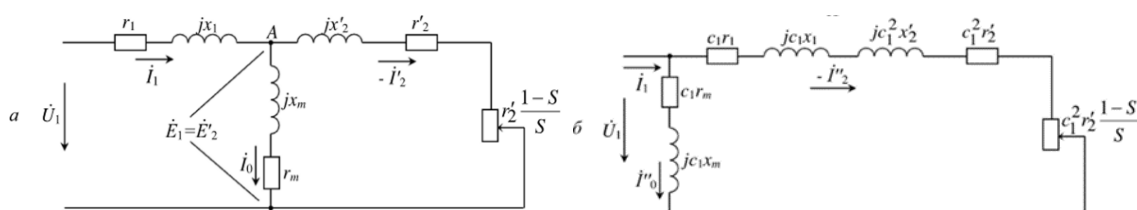


Figure 1. Substitution diagrams of induction motor: a) T-shaped diagram, b) G-shaped diagram

In contrast to the G-shaped diagram, the T-shaped diagram reflects the change of magnetising current when the frequency and load of the motor are changed. On the basis of the T-substitution diagram, it is possible to analyse the electromagnetic and mechanical characteristics at frequency control of the induction motor speed. To describe an idealised machine in rotating coordinate axes d, q, 0, taking into account the estimation of electromagnetic ( $M_{\text{e}}$ ) and braking ( $M_{\text{T}}$ ) moments, the system of the Park-Gorev equation [2] is applied:

$$\begin{aligned}
 u_d &= -\frac{d\psi_d}{dt} - \psi_d \frac{d\gamma}{dt} - R i_d; \\
 u_f &= \frac{d\psi_f}{dt} + R_f i_f; \\
 u_q &= -\frac{d\psi_q}{dt} + \psi_d \frac{d\gamma}{dt} - R i_q; \\
 0 &= \frac{d\psi_{1d}}{dt} + R_{1d} i_{1d}; \\
 u_0 &= -\frac{d\psi_0}{dt} - R i_0; \\
 0 &= \frac{d\psi_{1q}}{dt} + R_{1q} i_{1q}; \\
 \psi_d &= x_d i_d + x_{ad} i_f + x_{ad} i_{1d}; \\
 \psi_q &= x_q i_q + x_{aq} i_{1q}; \\
 \psi_f &= x_{aq} i_d + x_f i_f + x_{ad} i_d; \\
 \psi_{1d} &= x_{aq} i_q + x_{1q} i_{1q}.
 \end{aligned} \tag{1}$$

Here,  $i_a, i_b, i_c$  - stator phase currents;  $u_d, u_q$  - voltage at stator winding terminals along longitudinal and transverse axes;  $\psi_d, \psi_q, i_d, i_q$  - correspondence with stator winding flux-coupling and phase currents along longitudinal and transverse axes;  $R_1$  - active resistance of stator phase winding;  $u_f$  - voltage at field winding terminals;  $\psi_f, i_f$  - field winding flux-coupling and current;  $R_f$  - active resistance of field winding.

Further, the electromagnetic torque of the induction machine is determined according to

$$M_{\text{e}} = \frac{3}{2} (\psi_q i_d - \psi_d i_q). \tag{2}$$

The equation of motion of the electric drive is written in the form [3]

$$M_{\text{T}} - M_{\text{e}} = T_j \frac{d\omega}{dt} \tag{3}$$

The system of Park-Gorev equations reflects the simultaneous occurrence of electromagnetic and electromechanical transients, taking into account their mutual influence. In addition, for the most complete determination of the operating modes of the induction machine in static and dynamic modes, it is necessary to supplement this mathematical model with the moment of resistance  $M_c$  of the working mechanism, which will depend on the characteristic of the actuator. At present, many scientists carry out researches on improvement of mathematical models of induction machines. Among these works we can mention the following authors: V.O. Nagorny and A.V. Aristov obtained a simplified mathematical model of induction machine in the coordinate axes  $\alpha, \beta$  by excluding periodic windings in the stator [4]. The generalised model of the motor operating in the impulse mode of motion in the  $\alpha, \beta$  coordinate system is described with a system of differential equations. As a result of researches of Kupriyanov I.S., Belsky I.O. and Lukyanov A.V. [5] mathematical modelling of the induction machine at presence of a defect of squirrel cage of a rotor is received. Authors on the basis of spectral analysis of time signal proved that the presence of defects of squirrel cage rotor affects the amplitude of characteristic harmonics of phase currents in stator windings defects.

III. TO THE MATHEMATICAL MODEL OF A CENTRIFUGAL PUMP

Currently, the performance of most centrifugal pumping units is regulated by changing the hydraulic resistance of pipelines (network), which is inefficient and slow to respond. This system of pump performance regulation does not contribute to reducing production costs and increasing economic benefits. Therefore, many scientists are investigating the development and improvement of the mathematical model of centrifugal pump to build an energy-efficient performance regulation system.

To build a mathematical model of a centrifugal pump, we will use its laughing substitution, presented in Figure 2 [6].

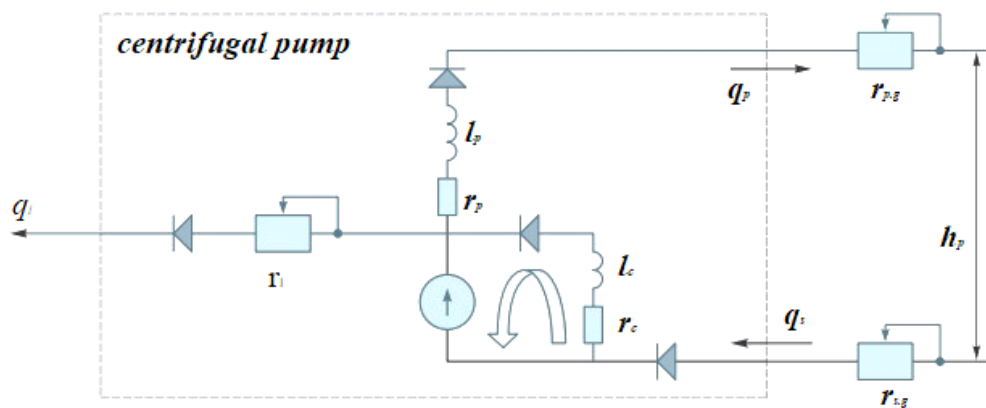


Figure 2. Centrifugal pump substitution scheme

$h_p, q_p$  - pressure and delivery of the centrifugal pump;  $q_s$  - flow rate in the suction connection of the pump;  $q_c, q_l$  - components of the fictitious pump delivery– circulation and leakage;  $r_{s,g}$  - resistance of the gate valve on the suction connection;  $r_c$  - resistance of the circulation circuit;  $r_l$  - resistance of the leakage circuit;  $l_p, l_c$  - delay lines of the pump volute and circulation circuit of the centrifugal pump;  $r_{p,g}$  - resistance of the pressure gate valve.

On the basis of the substitution scheme many authors have obtained mathematical models of centrifugal pumps [7-10]. In these works, the static Q-H characteristic of a centrifugal pump is approximated by a quadratic polynomial of the second order with a sufficient degree of accuracy

$$H(Q, \omega) = H_0 \left( \frac{\omega}{\omega_k} \right)^2 - RQ^2, \tag{4}$$

Where  $H_0$  - CP head in the mode of closed gate valve of the pressure main, fictitious no-load head at  $Q=0$ ;  $R$  - internal hydraulic resistance of CP;  $\omega, \omega_k$  - operating and catalog speed of CP;  $H_n, Q_n$  - nominal head and delivery of CP.

As experience of operation of centrifugal pumps shows, the main factor of influence on the operation mode is the resistance torque on the motor shaft, which is determined according to

$$M_c = \frac{\rho \cdot g \cdot Q \cdot H(Q, \omega)}{\omega \cdot \eta(Q)}, \tag{5}$$



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where  $\rho$  - density of the pumped liquid;  $\omega$  - angular speed of the CN motor;  $Q, H(Q, \omega), \eta(Q)$  - delivery, head and efficiency of the pump operating at speed .

In accordance with equation (5), the resistance moment is also influenced by process parameters.

## IV. CONCLUSION

Thus, to build a mathematical model of a frequency-controlled induction motor, we can use a system of equations in rotating coordinate axes  $d, q$ , while taking into account the moment of corotation of the  $M_c$  of the actuator of the electric drive. To build a mathematical model of the centrifugal pump, it is necessary to take into account the process parameters, including the density of the pumped liquid, head, pressure, height and the required capacity of the pumping unit.

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